1. Sequences

1, 2, 3, . . .

duck, duck, goose, . . .

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4, 3, 2, 1
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Sequences: Notes and Vocabulary

A **sequence** is an ordered list of things.

Here is a sequence of numbers: 1, 2, 3, 4, 5
Here is a sequence of letters: A, B, C, D
Here is a different sequence of numbers: 5, 4, 3, 2, 1

The "things" in the sequence are called **terms**. In the sequence 5, 4, 3, 2, 1

- first term is 5
- the third term is 3
- the fifth term is 1.

What is the first term of the sequence 1, 2, 3, 4, 5? _____

What is the fourth term of the sequence A, B, C, D? _____

A sequence can be finite or infinite. The sequences above are all finite.

Here's another example of a finite sequence: 10, 9, 8

Here's another example: 400, 500, 600, 700, 800

Write down your own example of a finite sequence: _____

An infinite sequence is a list that goes on forever. It has an infinite number of terms. We show that it goes on forever by three periods in a row at the end: . . .

Another name for "three periods in a row" is ellipsis.

Here is an example of an infinite sequence: 1, 2, 3, . . .

Another example of an infinite sequence: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4} \ldots$

Write down your own example of an infinite sequence: _____

A sequence of numbers might follow a pattern. If you know the pattern, you can figure out what the next term is. Can you write the next few terms of these sequences?

2, 4, 6, _____ , _____ , _____ , …

4, 7, 10, _____ , _____ , _____ , …

Some sequence patterns might be more challenging, such as this finite sequence:

3, 2, 1, 2, 3, 3, 3, 2, 2, 2, 3, 5, 5, … , 1

Can you fill in the missing terms? Think for a minute. A hint for this sequence is at the bottom of this page.

You can also use an ellipsis to write down a missing set of terms in a finite sequence.
For example: 1, 2, 3, … , 10
Is a short way to write 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
In the example 1, 2, 3, … , 10 the ellipsis stands for the missing terms 4, 5, 6, 7, 8, 9

In the following finite sequences, what are the missing terms?

Sequence	Missing Terms
10, 9, 8, … , 4	
0, 3, 6, … , 24	
77, 72, 67, … , 47	

Hint for the sequence 3, 2, 1, 2, 3, 3, 3, 2, 2, 2, 3, 5, 5, … , 1 above: musical notes. The sequence corresponds to a melody. If you don't have musical experience, this example will not mean much to you.

In that case, try to figure out the following pattern:

0, 3, 8, 15, 24, 35, 48, . . .

1-1. Sample Sequences

Here are some sequences. What are the next three terms?

a. 5, 5.3, 5.6, 5.9, 6.2, . . .

b. 2, 6, 18, 54, 162, . . .

c. 2, 5, 9, 14, 20, . . .

d. one, one thousand, one million, . . .

1-2. Consecutive Terms

Consecutive terms are terms that exactly follow one another in a sequence.
In the sequence 1, 2, 3, … the terms 3, 4, 5, 6, and 7 are consecutive terms.
In the sequence A, B, C, … , Z the terms P and Q are consecutive terms.

Write three consecutive whole numbers: _____, _____ , _____

Write three consecutive whole numbers bigger than a million:

_____ , _____ , _____

Write three consecutive odd numbers: _____, _____ , _____

Write three consecutive multiples of five: _____ , _____ , _____

Write three consecutive prime numbers: _____ , _____ , _____

Write three consecutive two-digit primes: _____ , _____ , _____

Write three consecutive perfect squares: _____ , _____ , _____

Pose a more challenging problem involving consecutive numbers that you are pretty sure you are able to solve. Make it as different from the above problems as you can.

Example: Write three consecutive four-digit multiples of 17.

Pose an even more challenging problem involving consecutive numbers that you are pretty sure you are not able to solve (yet).

Example: What are the dates of the next three consecutive supernova explosions that will happen in the Milky Way?

1-3. Fibonacci Sequence

The Fibonacci sequence is: 1, 1, 2, 3, 5, 8, 13, . . .

a. The terms of the Fibonacci sequence follow a pattern. Describe the pattern.

b. List the next three terms of the Fibonacci sequence.

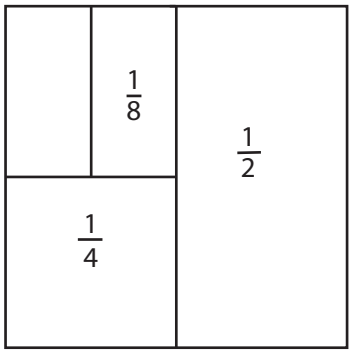
c. The terms of the Fibonacci sequence are called Fibonacci numbers. How many prime Fibonacci numbers can you find?

d. Choose any three consecutive terms of the Fibonacci sequence. Square the middle term and multiply the two other terms. Compare the results. Repeat this process with other groups of three consecutive terms. What is the pattern? (This is Cassini's Identity.)

1-4. Zeno's Paradox In A Box

The sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

Is illustrated in this box:

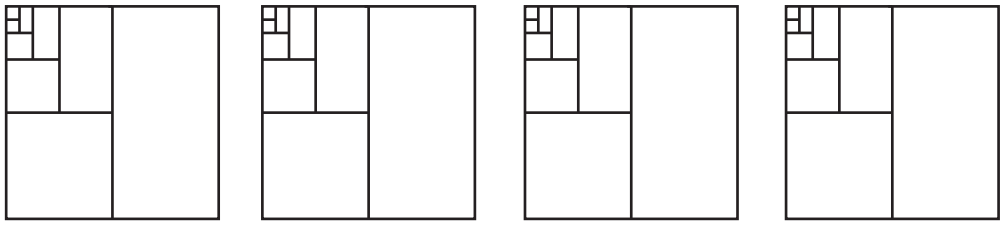


Draw in a few more terms in the box.

Describe what happens to the terms of the sequence, as they go on and on.

Here is another sequence: $\frac{1}{2}, \ \frac{1}{2}+\frac{1}{4}, \ \frac{1}{2}+\frac{1}{4}+\frac{1}{8}, \ldots$

Illustrate this sequence in these boxes:



Describe what happens to the terms of this sequence, as they go on and on.
If you know how to add fractions, work out the terms and see if they follow a pattern.

1-5. Days Of The Week

a. What day of the week is today? _____

b. What day of the week will it be 100 days from now? _____

c. 500 days from now, will it be a different day of the week from today?

d. Name a three-digit number for which you know that it will be the same day as today, that many days from now.

1-6. Counting Terms

a. How many terms are in the sequence: 1, 3, 5, 7, . . . , 993, 995, 997, 999 ?

b. Find the 1000^{th} term in the sequence: 2, 3, 4, 5, 6, . . .

c. Find the 1000^{th} term in the sequence: 2, 4, 6, 8, . . .

1-7. Cumulative Peanuts

Ella the elephant ate 1000 peanuts from Monday through Friday. Each day she ate 10 peanuts more than the previous day. How many peanuts did she eat on Friday?

1-8. Amoebas

An amoeba reproduces by dividing into two. Suppose it takes a day for an amoeba to eat enough food to be able to divide, and a scientist drops an amoeba into a pond in his back yard on January 1.

a. Write down the sequence of amoeba population values for January 1, January 2, and so on.

b. The scientist notices that the pond is full of amoebas on February 10.
On what date would the pond be full if he dropped two amoebas instead of one on January 1?

1-9. Trina's Sequence Machine

Trina has a sequence machine. She gives it a first number and the machine manufactures the next term, according to these rules:

Rule 1: If the number is less than 10, add 3 to get the next term.
Rule 3: If the number is equal to 10, subtract 5 to get the next term.
Rule 4: If the number is greater than 10, subtract 6 to get the next term.

a. If the first term is 9, what is the 100^{th} term?

b. If the first term is 2, what is the 100^{th} term?

1-10. Julio's Sequence Machine

Julio has a sequence machine. He gives it a first number and it manufactures the next terms according to these rules:

Rule 1: If the number is greater than 8, subtract 7 to get the next term.
Rule 2: If the number is less than or equal to 8, double it to get the next term.

Work out the sequences for various different starting numbers.

First Number	Julio's Sequence
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

What patterns do you see?

Each sequence is different, because it has a different first term, but are some of Julio's sequences similar to each other, or related to each other?

What's a good way to define *similar sequence* in the world of Julio's sequences?

How To Deal With Sequence Problems

Learn the vocabulary:

Look for patterns such as:

There might be surprises such as: